

# Model-Based Active Exploration

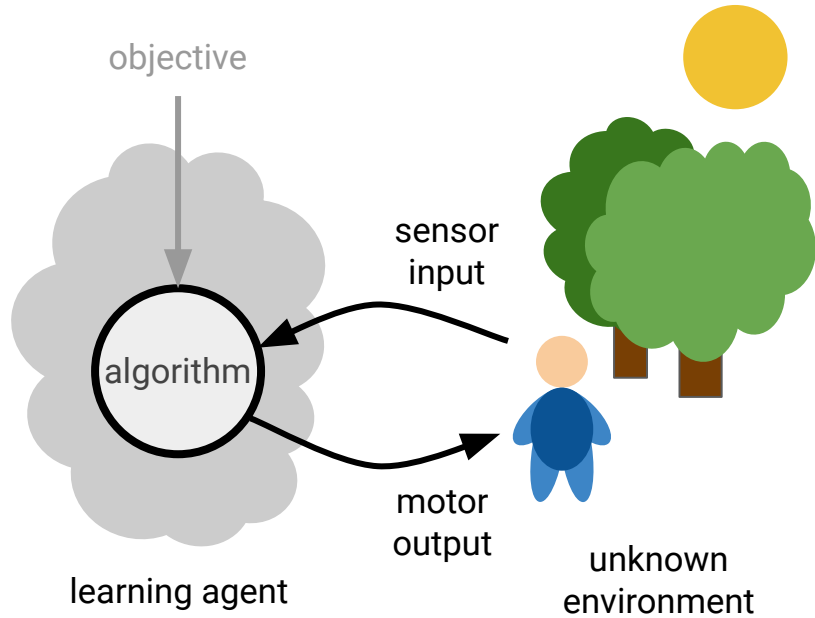
Pranav Shyam, Wojciech Jaskowski, Faustino Gomez



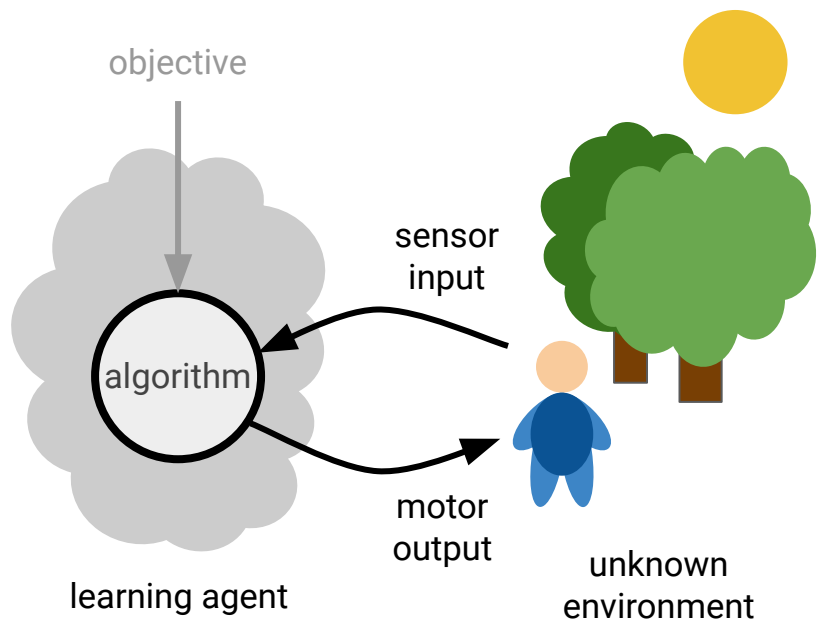
[arxiv.org/abs/1810.12162](https://arxiv.org/abs/1810.12162)

Presentation by Danijar Hafner

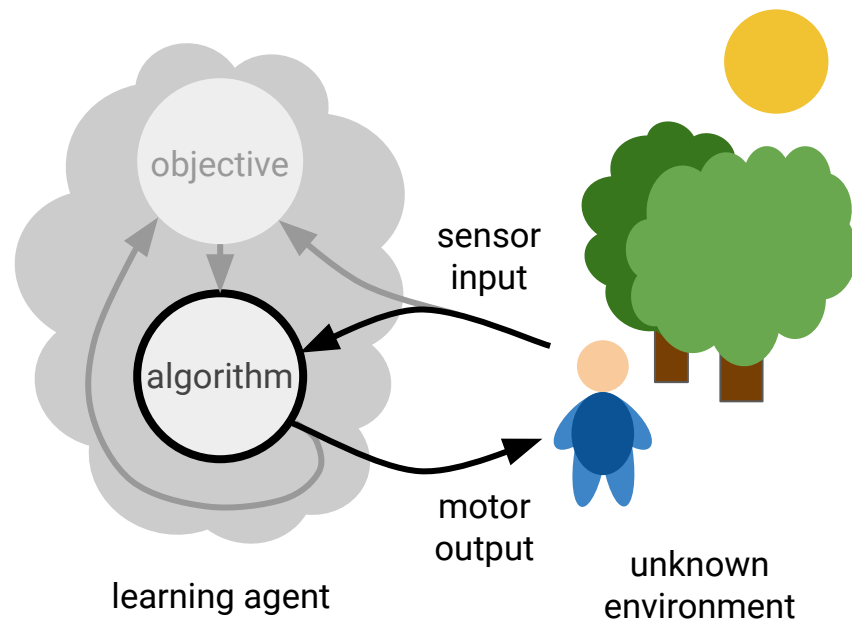
# Reinforcement Learning



# Reinforcement Learning



# Intrinsic Motivation



# Many Intrinsic Objectives

- Information gain e.g. [Lindley 1956](#), [Sun 2011](#), [Houthoofd 2017](#)
- Prediction error e.g. [Schmidhuber 1991](#), [Bellemare 2016](#), [Pathak 2017](#)
- Empowerment e.g. [Klyubin 2005](#), [Tishby 2011](#), [Gregor 2016](#)
- Skill discovery e.g. [Eysenbach 2018](#), [Sharma 2020](#), [Co-Reyes 2018](#)
- Surprise minimization e.g. [Schrödinger 1944](#), [Friston 2013](#), [Berseth 2020](#)
- Bayes-adaptive RL e.g. [Gittins 1979](#), [Duff 2002](#), [Ross 2007](#)

# Information Gain

Without rewards, the agent can only learn about the environment.

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Need to represent uncertainty about  $W$  to tell how much we have learned.



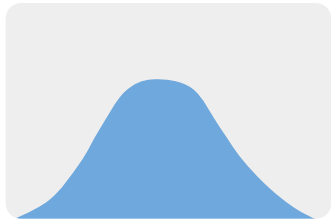
$p(W)$

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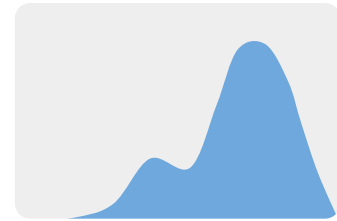
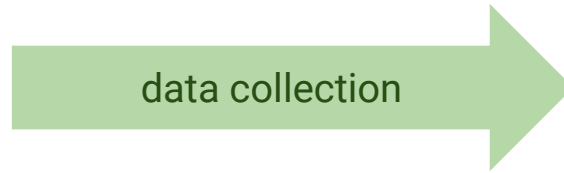
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$p(W)$



$p(W | X)$

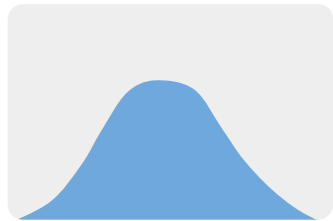


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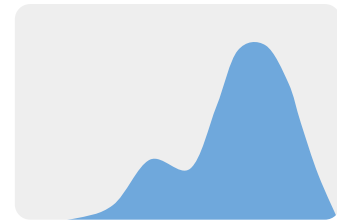
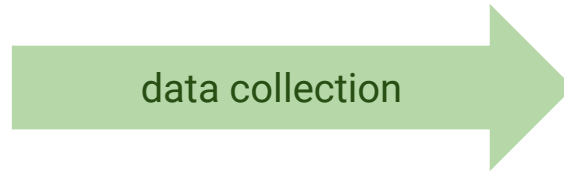
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To gain the most information, we aim to maximize the mutual information between future sensory inputs  $X$  and model parameters  $W$ :

$$\max_a I(X; W | A=a)$$

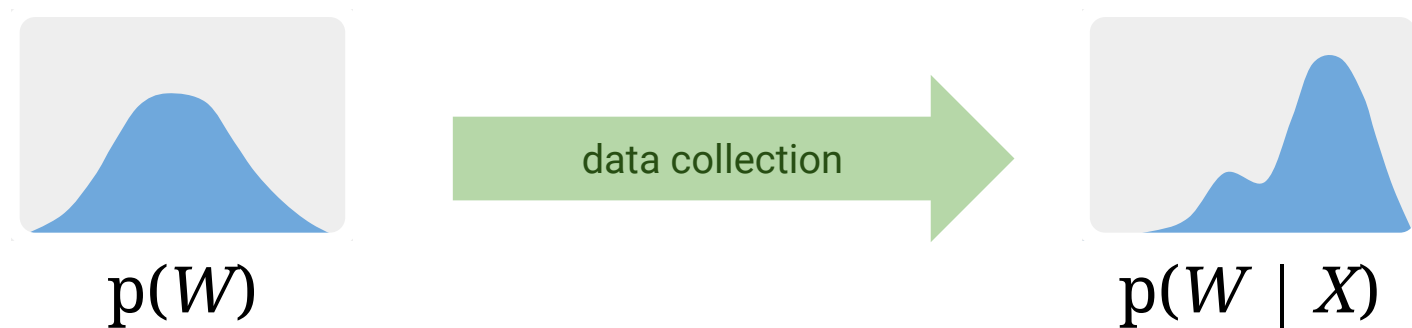
Both  $W$  and  $X$  are random variables

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$$\max_a I(X; W | A=a) = ?$$

Both  $W$  and  $X$  are random variables

# Retrospective Infogain

e.g. VIME, ICM, RND

$$\text{KL}[p(W | X, A=a) || p(W | A=a)]$$

Collect episodes, train world model, record improvement, reward the controller by this improvement

Infogain depends on agent's knowledge that keeps changing, making it a non-stationary objective

The learned controller will lag behind and go to states that were previously novel but are not anymore

# Expected Infogain

e.g. MAX, PETS-ET, LD

$$I(X; W | A=a)$$

Need to search for actions that will lead to high information gain without additional environment interaction

Learn a forward model of the environment to search for actions by planning or learning in imagination

Computing the expected information gain requires computing entropies of a model with uncertainty estimates

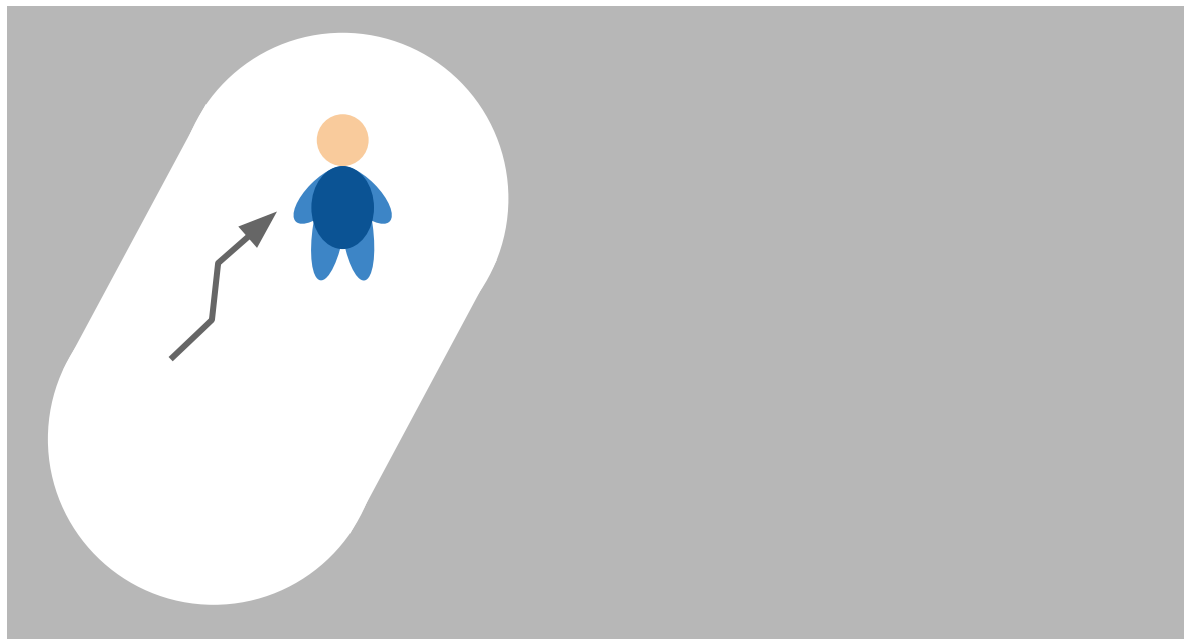
# Retrospective Novelty



Episode 1

Everything unknown

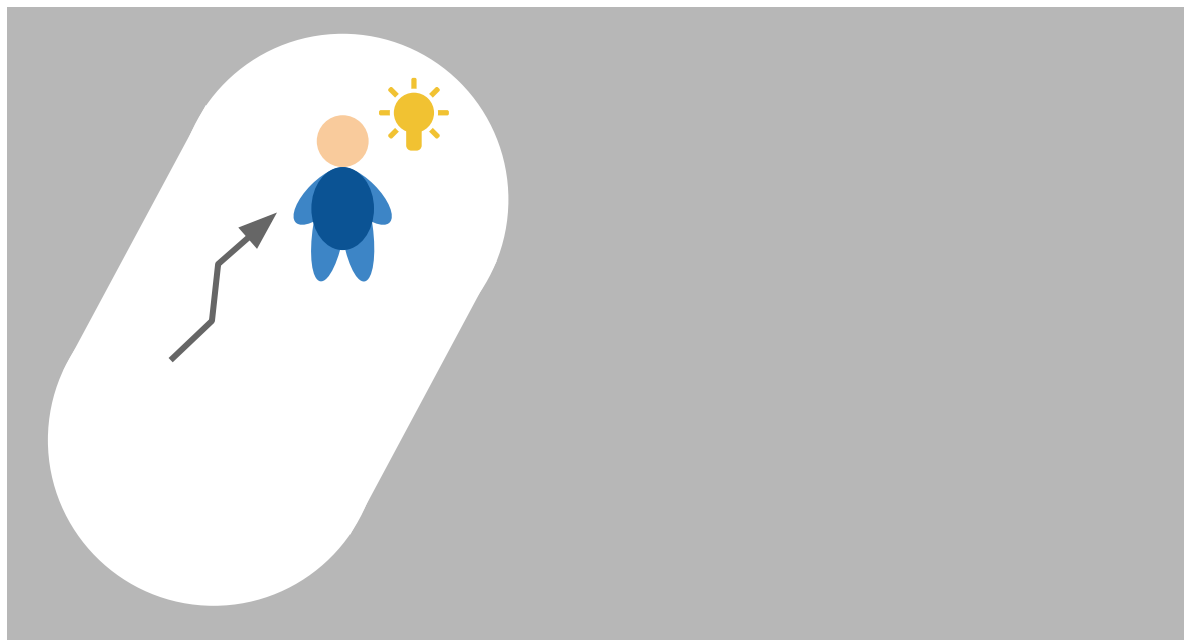
# Retrospective Novelty



Episode 1

Random behavior

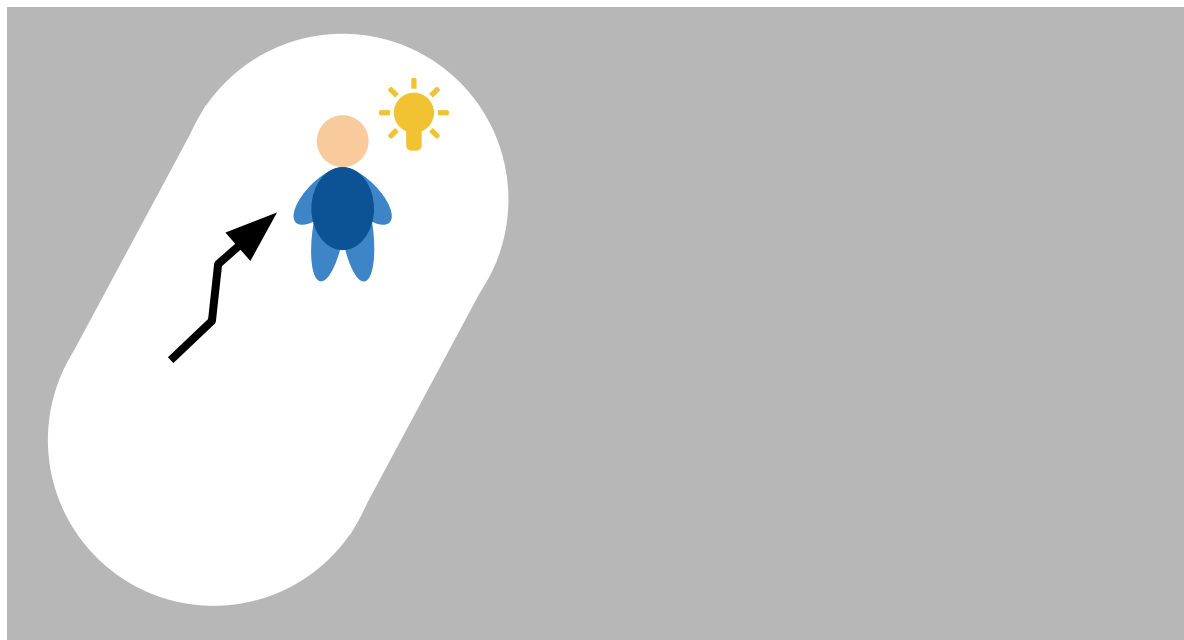
# Retrospective Novelty



Episode 1

High novelty

# Retrospective Novelty



Episode 1

Reinforce behavior

# Retrospective Novelty



Episode 2

Repeat behavior



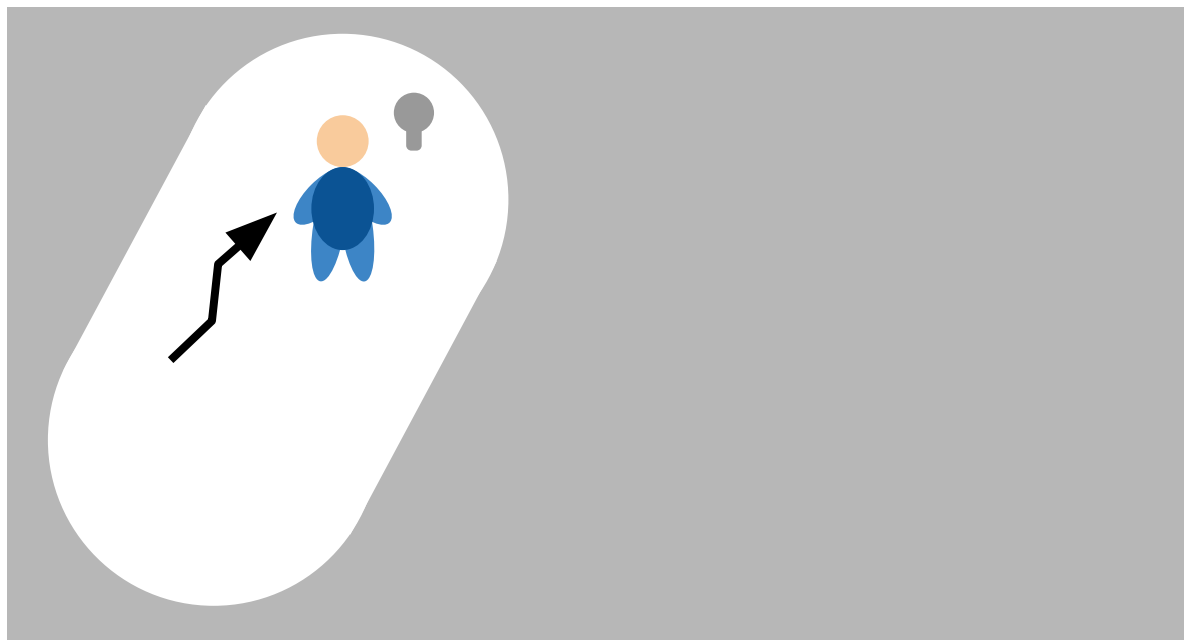
# Retrospective Novelty



Episode 2

Reach similar states

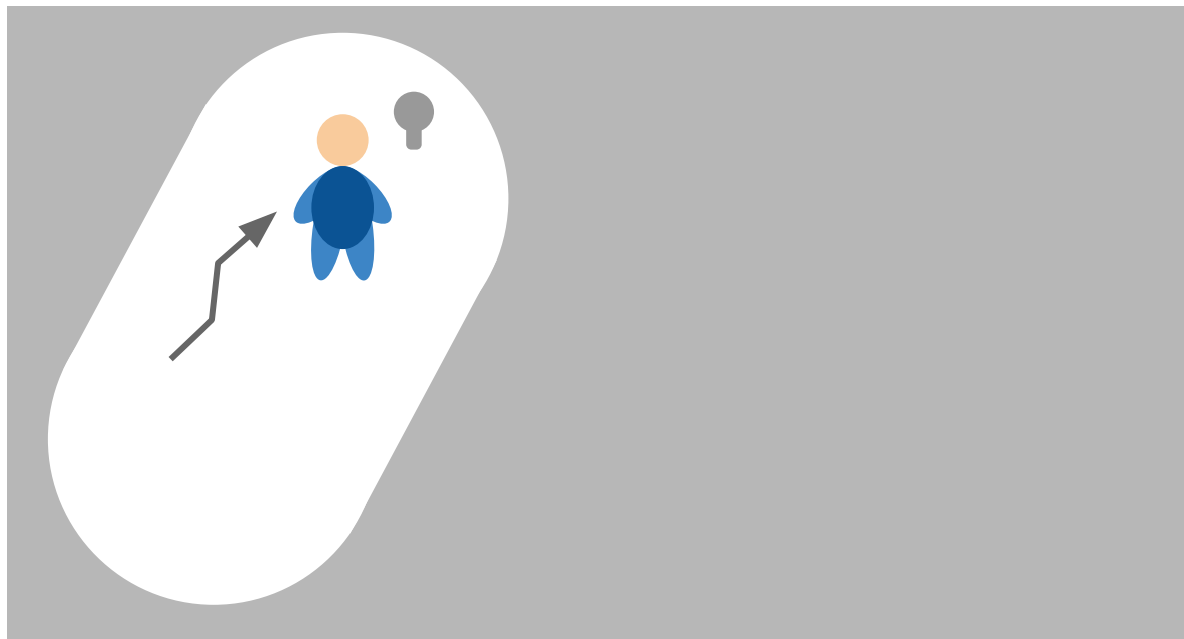
# Retrospective Novelty



Episode 2

Not surprising anymore :(

# Retrospective Novelty



Episode 2

Unlearn behavior

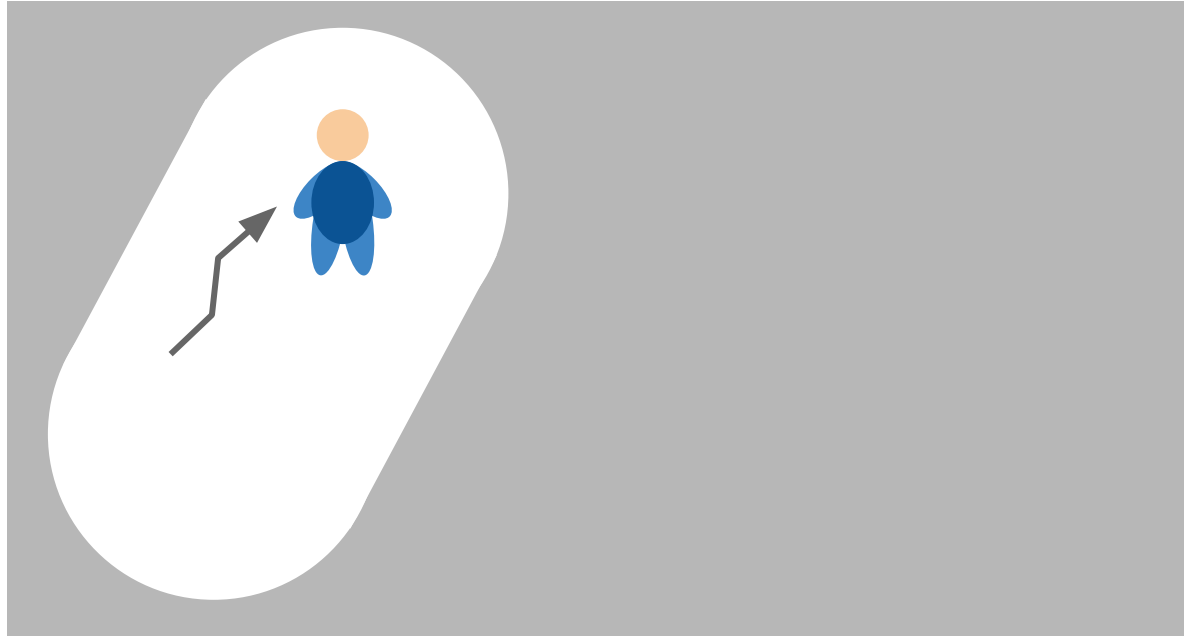
# Retrospective Novelty



Episode 3

Repeat behavior

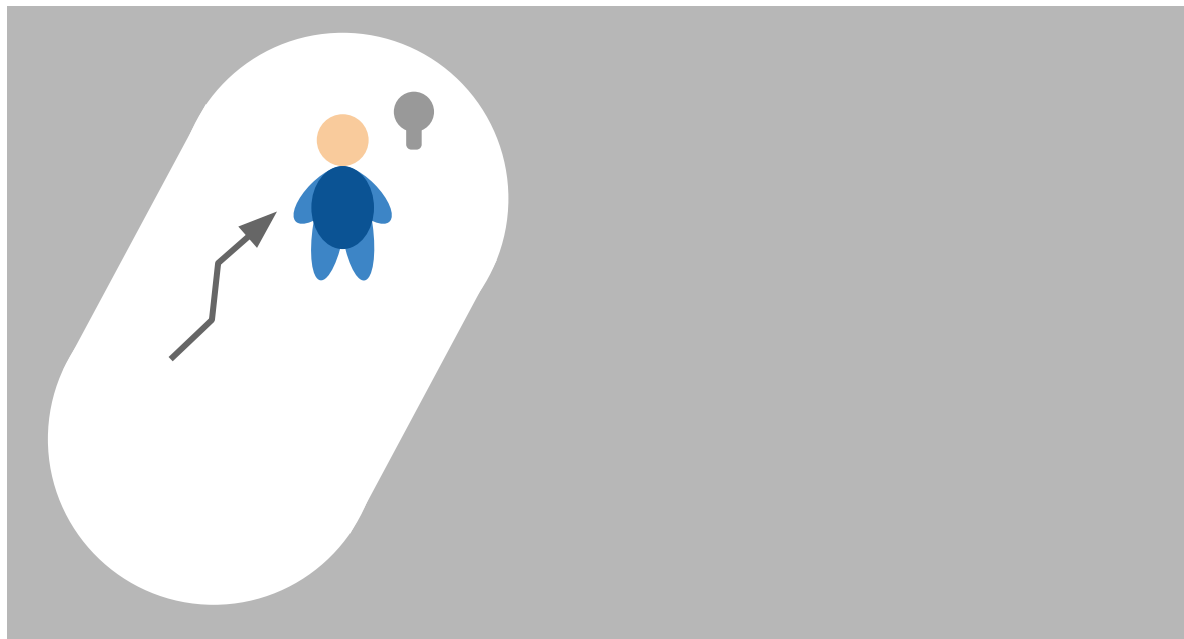
# Retrospective Novelty



Episode 3

Repeat behavior

# Retrospective Novelty



Episode 3

Still not novel

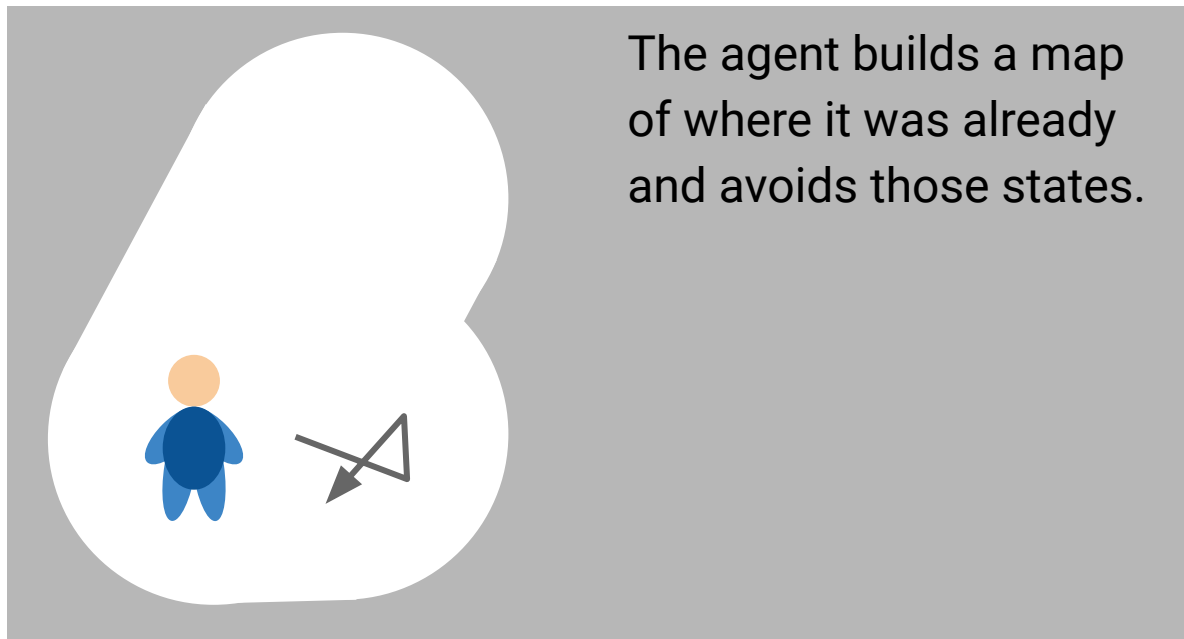
# Retrospective Novelty



Episode 3

Unlearn behavior

# Retrospective Novelty



Episode 4

Back to random behavior



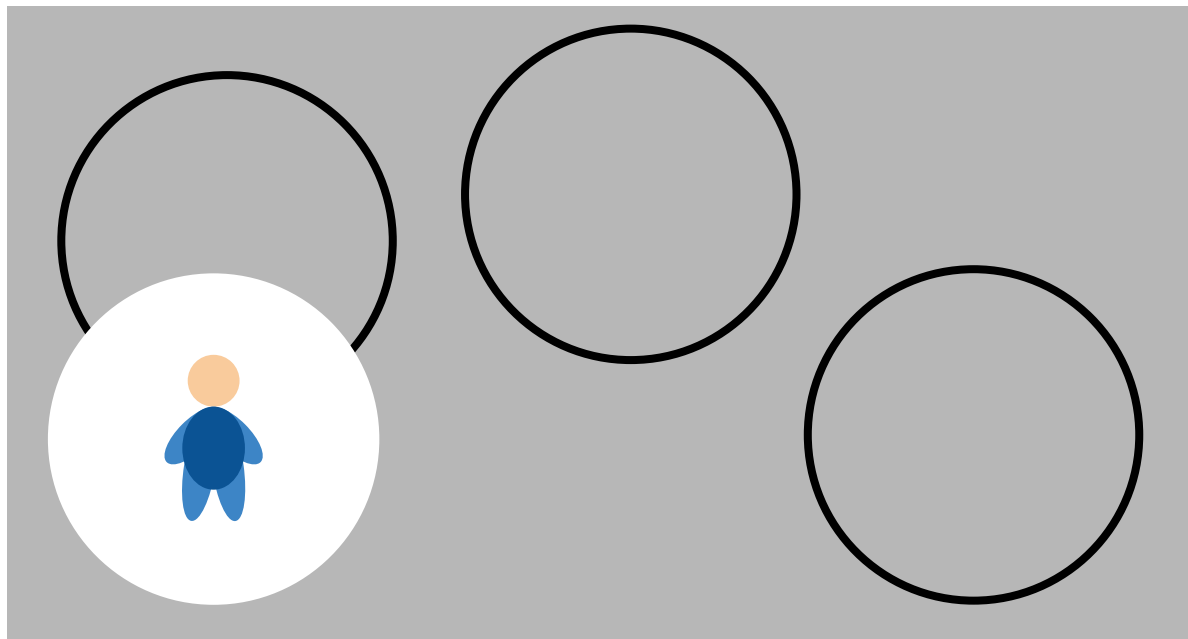
# Expected Novelty



Episode 1

Everything unknown

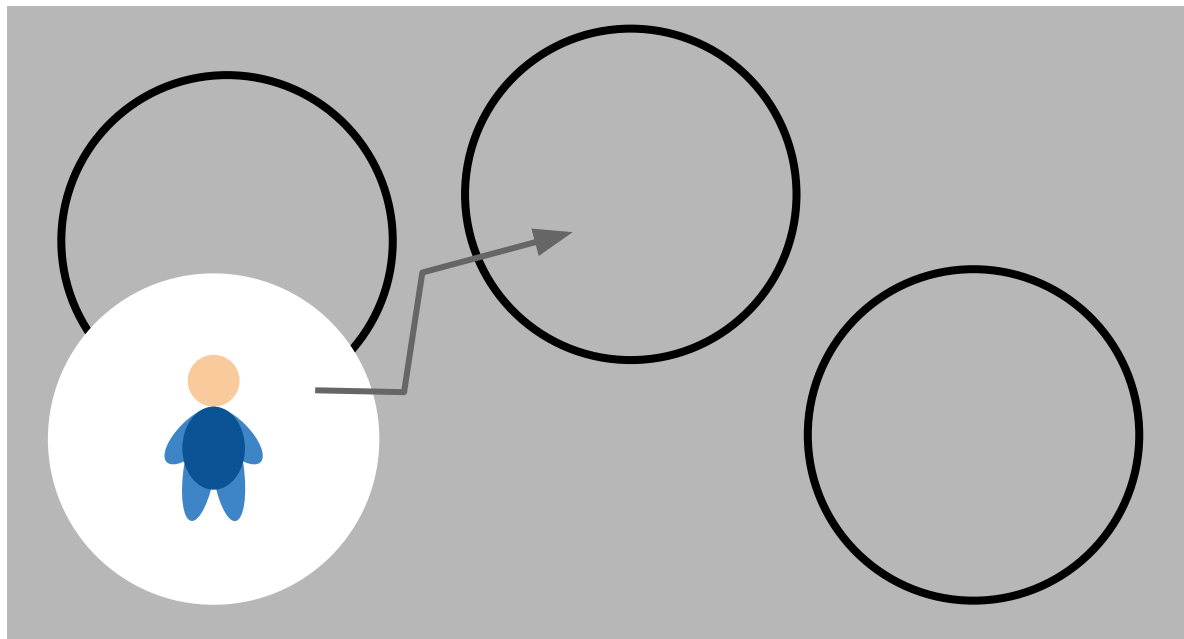
# Expected Novelty



Episode 1

Consider options

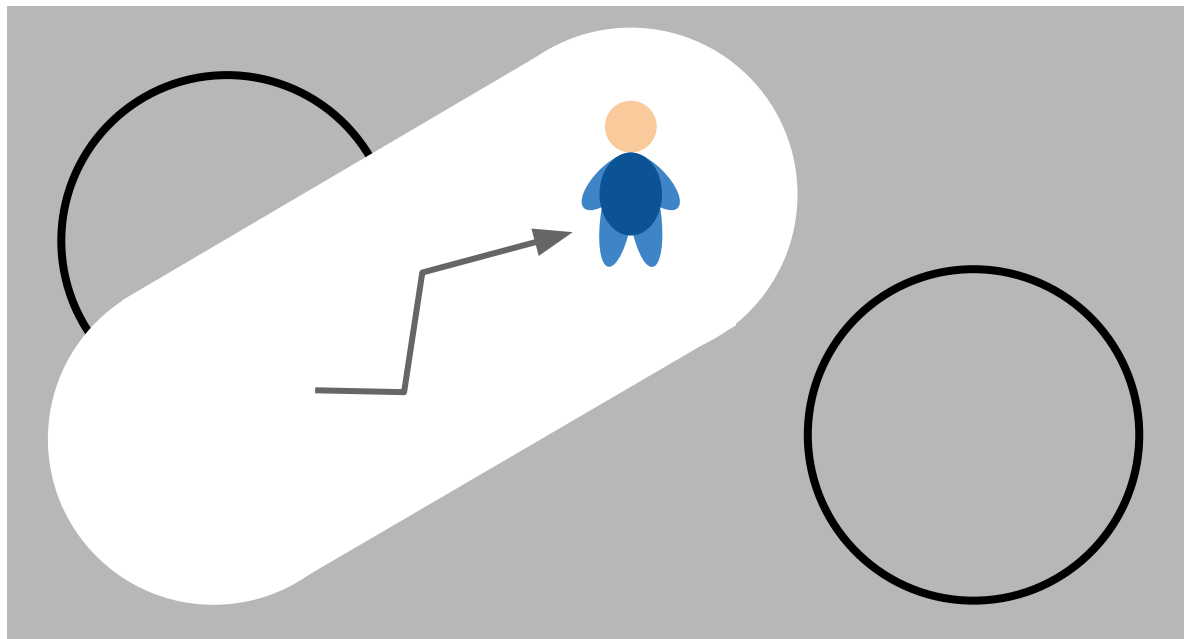
# Expected Novelty



Episode 1

Execute plan

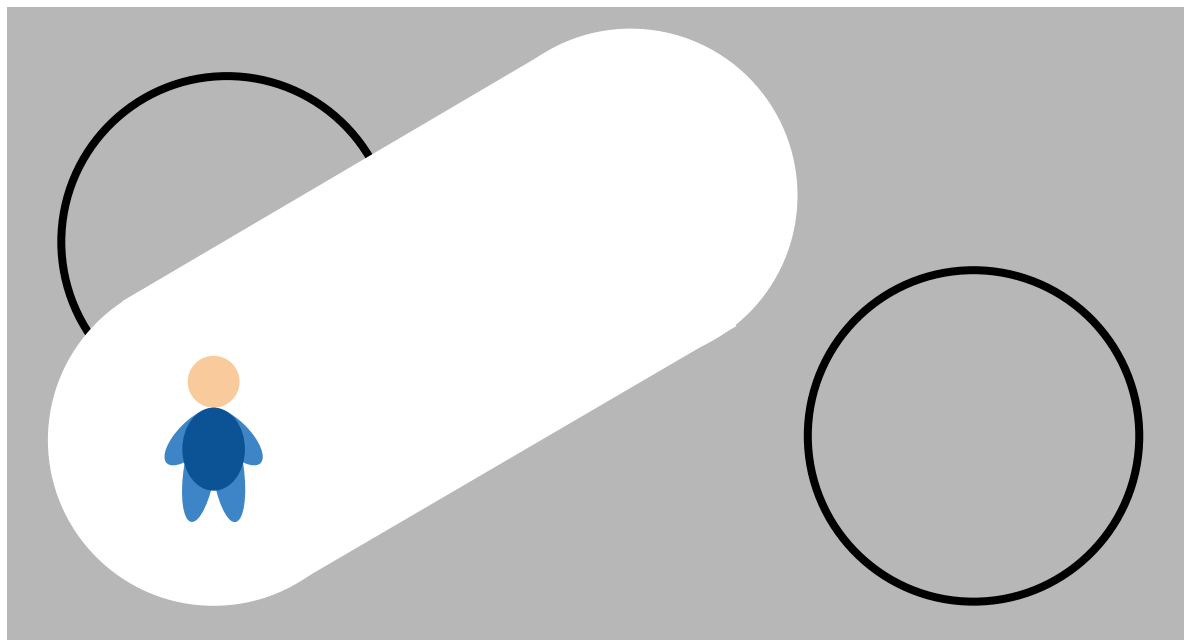
# Expected Novelty



Episode 1

Observe new data

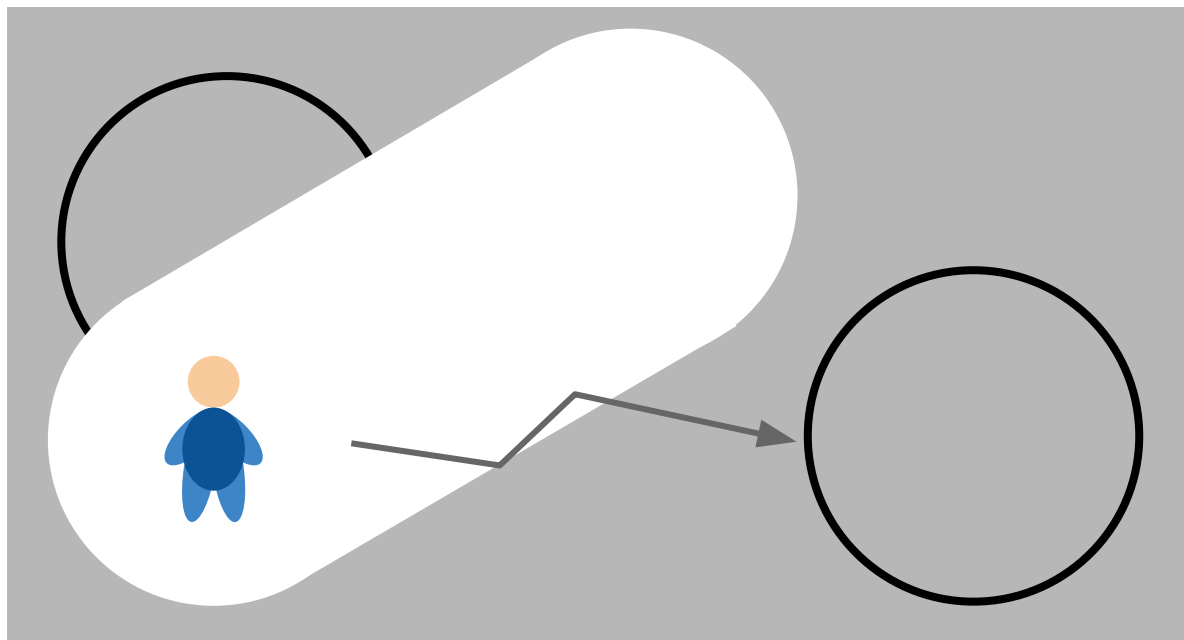
# Expected Novelty



Episode 2

Consider options

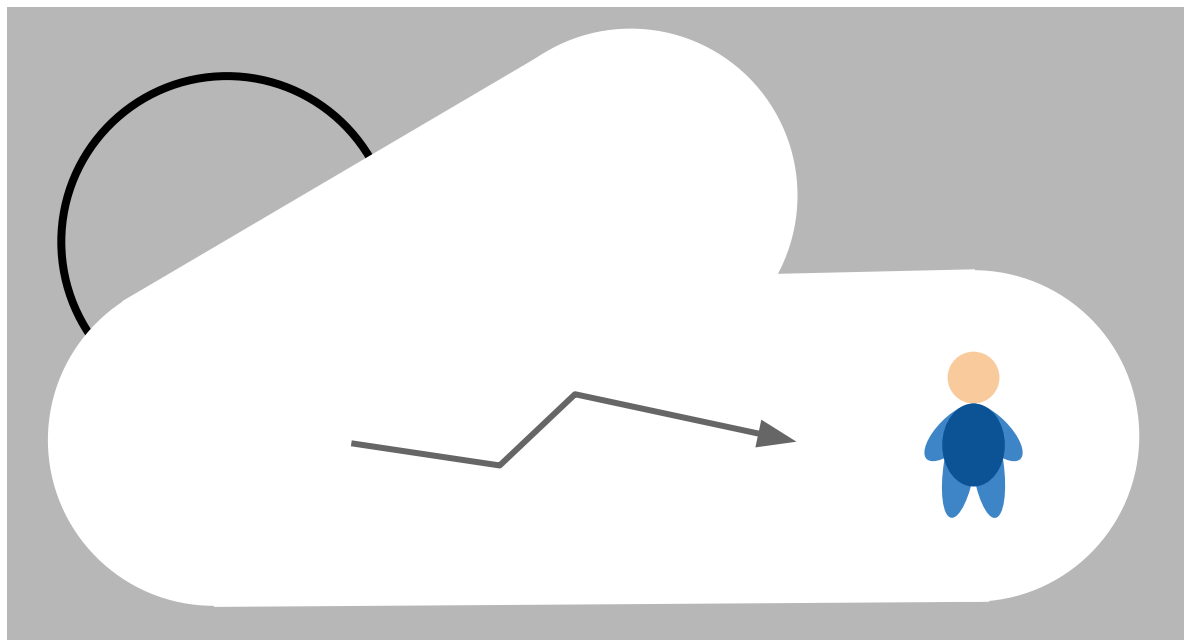
# Expected Novelty



Episode 2

Execute plan

# Expected Novelty



Episode 2

Observe new data

# Ensemble of Dynamics Models

Learn dynamics both to represent knowledge and to plan for expected info gain



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epistemic uncertainty      total uncertainty      aleatoric uncertainty

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Information gain targets uncertain trajectories with low expected noise

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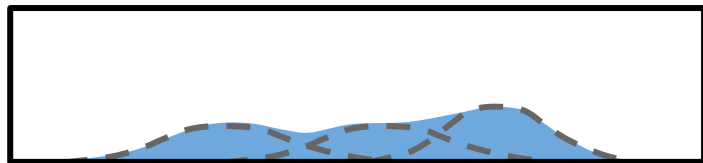
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Overlapping modes means less total uncertainty

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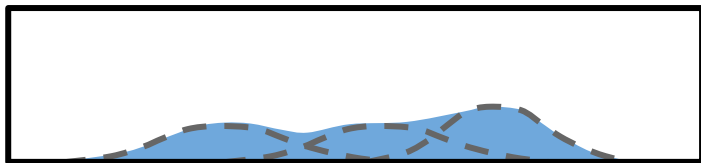
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epistemic uncertainty

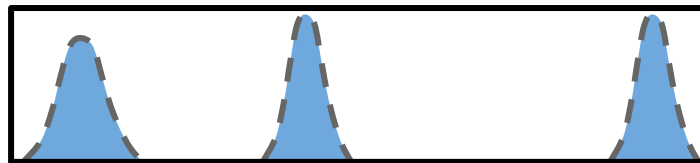
total uncertainty

aleatoric uncertainty

Information gain targets uncertain trajectories with low expected noise



Wide predictions mean high expected noise  
Overlapping modes means less total uncertainty



Narrow predictions mean low expected noise  
Distant modes means large total uncertainty

# Expected Infogain Approximation

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Total uncertainty:           $H(1/K \sum p(X \mid W=w_k, A=a))$

Gaussian entropy has a closed form, so we can compute the aleatoric uncertainty.  
GMM entropy does not, sample it or switch to Renyi entropy that has a closed form.

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**Algorithm 1** MODEL-BASED ACTIVE EXPLORATION

---

**Initialize:** Transitions dataset  $D$ , with random policy

**Initialize:** Model ensemble,  $\tilde{T} = \{t_1, t_2, \dots, t_N\}$

**repeat**

**while** episode not complete **do**

    ExplorationMDP  $\leftarrow (\mathcal{S}, \mathcal{A}, \text{Uniform}\{\tilde{T}\}, u, \delta(s_\tau))$

$\pi \leftarrow \text{SOLVE}(\text{ExplorationMDP})$

$a_\tau \sim \pi(s_\tau)$

    act in environment:  $s_{\tau+1} \sim \mathcal{P}(\mathcal{S} | s_\tau, a_\tau, t^*)$

$D \leftarrow D \cup \{(s_\tau, a_\tau, s_{\tau+1})\}$

    Train  $t_i$  on  $D$  for each  $t_i$  in  $\tilde{T}$

**end while**

**until** computation budget exhausted

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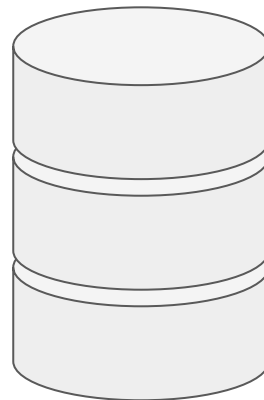
# Compared Algorithms



**Learning from imagined trajectories (Expected)**

MAX: JSD infogain

TVAX: State variance

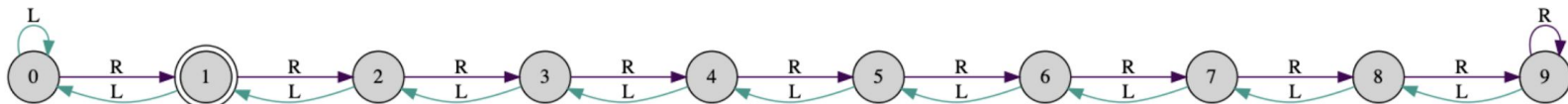


**Learning from experience replay (Retrospective)**

JDRX: JSD infogain

PERX: Prediction error

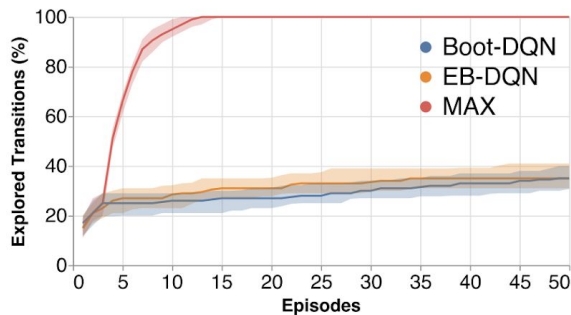
# Exploration Chain Domain



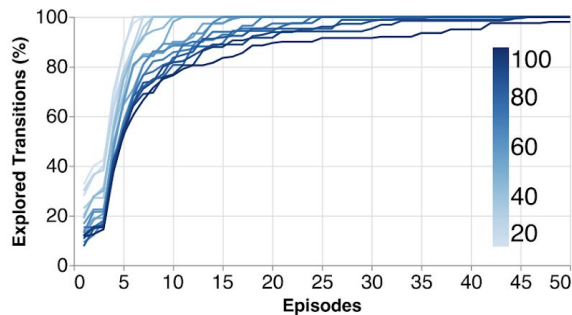
+0.001

(a) Chain Environment of length 10.

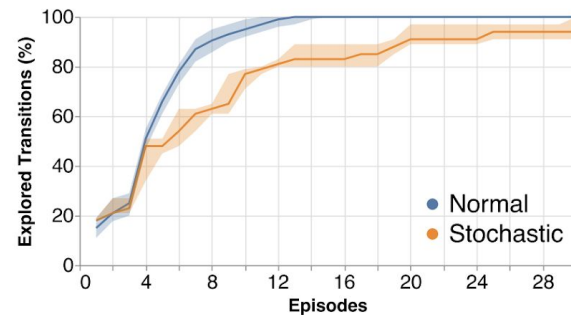
+1



(b) 50-state chain.

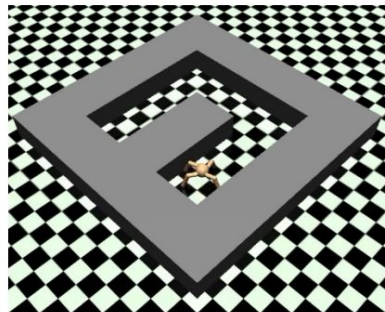


(c) Chain lengths

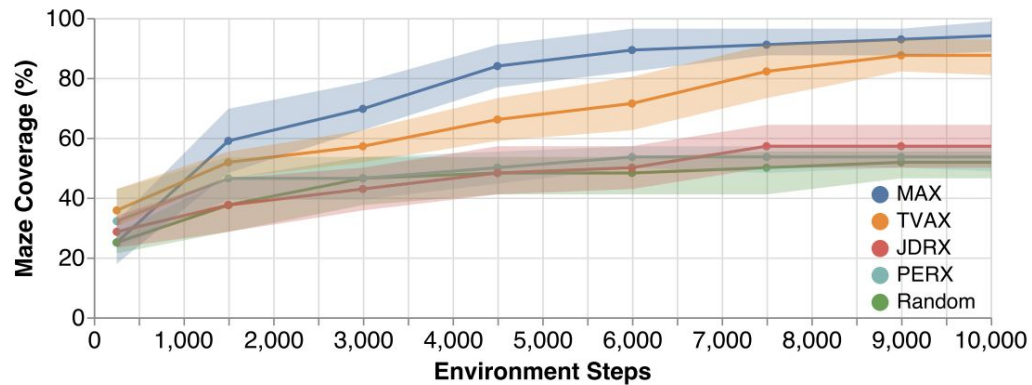


(d) Stochastic trap

# State coverage of Ant Maze



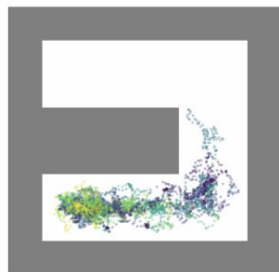
(a) Ant Maze Environment



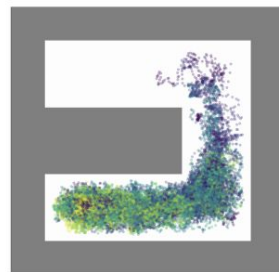
(b) Maze Exploration Performance



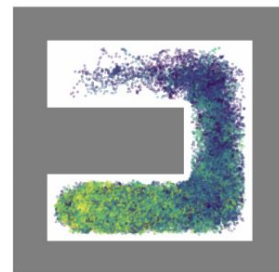
(c) 300 steps



(d) 600 steps



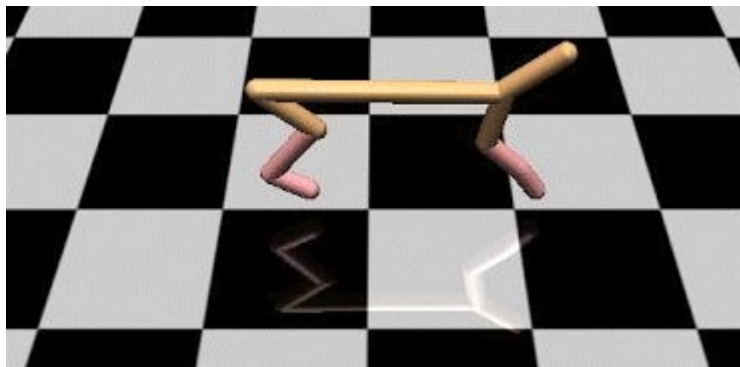
(e) 3600 steps



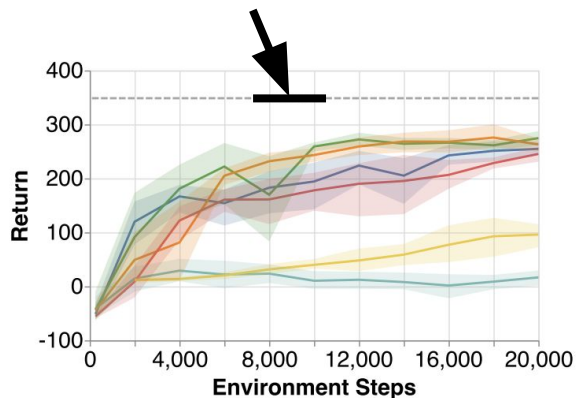
(f) 12000 steps

# Zero-Shot Adaptation

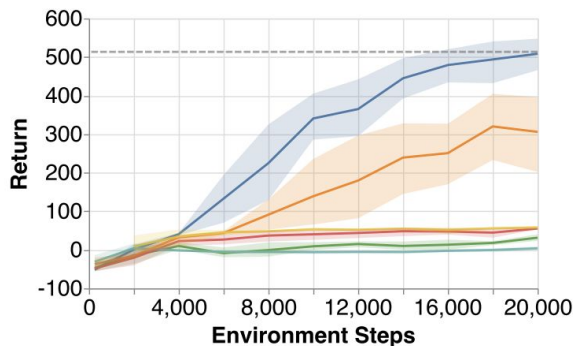
Learn evaluation policy inside of learned model given a known reward function



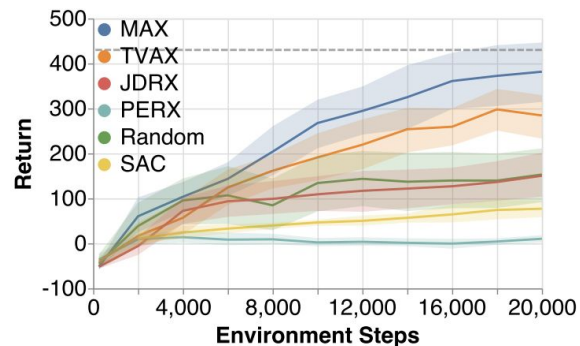
model-free with 10x data



(a) Running task performance  
no exploration needed



(b) Flipping task performance  
exploration needed



(c) Average performance

# Conclusions

Information gain is a principled task-agnostic objective

As a non-stationary objective, it should be optimized in expectation

This requires a dynamics model for planning to explore

Ensemble of Gaussian dynamics is a practical way to represent uncertainty

